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A CASE STUDY ON APPLICATION OF TRANSPORTATION PROBLEM

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Abstract

A transportation problem always concerned with defining an optimal strategy for distributing several supply centers to minimize the cost and time. Transportation problem is widely used as a decision-making tool in various fields. A company desires for maximum profit and minimize any cost. Among all of the costs, transportation cost has seen as major issue. So, it has become a priority for companies to minimize the transportation cost. Here, a study is done on one company which was left unsolved in the case study "A Case Study on Application of Transportation Problem of some Select Companies" conducted by Ahmed and Kalita. In this study, north-west corner rule, least entry method, Vogel's Approximation methods are applied to get the minimum cost. Optimality is tested and compared the results that are solved by Simplex method using linprog solver of Matlab2016a.

Keywords: transportation problem; case study; optimality test; linprog solver.

1. Introduction

1.1 Introduction to Transportation problem:

About 6% of the GDP is due transportation cost in India. Transportation itself costs 10.4% of the total revenue of the US businesses. Transportation problem deals with transporting products or particulars to one destination from another destination. It also investigates the cost due to transportation. principally, transportation problem studies transportation of products or particulars stored at several sources to different destinations. Transportation problem in Operations Research has wide operations in force control, product planning, scheduling, particular allocation and so on. In 1781, French mathematician Gaspard Monge homogenized transportation problem. The first one to study the transportation mathematically was A.N Tolstoy, in 1920's. The conception of transportation problem or the proposition developed after the discovery of operation exploration first came into picture in world war I when wisdom was applied to ameliorate military operations. In a transportation problem, first each source is associated with different possible destinations. Total movement from each source towards each destination is given. It's asked to determine how the association be made subject to the limitations on total.

There are three methods for solving transportation problem - north west corner method, least cost method and Vogel's approximation method.

North-West Corner Method:

This above-mentioned method starts at the northwest corner cell in transportation table.

- First, we opt the northwest cell in transportation table.
- Allocate as much as feasible in that cell until the demand of the first column or the supply of the first row is satisfied.
- If the demand is fulfilled, move to the cell horizontally to the right located at second column and allocate as much as possible.
- If the supply is fulfilled, then move to the cell vertically down located at second row and allocate as much as possible.
- If both supply and demand are fulfilled, then move one cell diagonally and allocate as much as possible.
- This procedure will persist until all the allocations are over.

Least cost method:

In this method the cheapest route is allocated among all the routes.

- Determine the cell with the least (minimum) cost in the transportation table.
- Allocate the maximum doable volume to the cell.
- Exclude the row or column where an allocation is made.
- Repeat the above steps until all the allocations are made

Vogel's Approximation Method:

Vogel's Approximation Method produces a better solution compare to least cost method and northwest corner method.

- Calculate the penalties for each row and each column.
- Select the column or row with the biggest penalty.

- In the named row or column, allocate the maximum doable volume to the cell with the minimal cost.
- cross out those row or column where all the allocations are made.
- Repeat the procedure until all the allocations are covered.

1.2 Mathematical Form of Transportation Problem

Let us consider that m be the number of sources and n be the number of destinations. Let p_i be the number of supply units available at source and q_j be the number of demand units required at the destinations where $i = 1, 2, \dots m,\ j = 1,2, \dots n$.

Let z_{ij} refer to the unit transportation problem for transporting. $x_{ij>0}$ be the number of units shipped from source i to destination j. Then the LP problem is as follows:

$$\text{Minimize } Z=\sum_{i=1}^{m} \sum_{i=1}^{n} y_{ij}\ x_{ij}$$

$$\text{Subject to } \sum_{i=0}^{n} x_{ij} = p_i$$

$$\sum_{i=0}^{m} y_{ij} = q_j$$

The system needs to be balanced. it is a necessary and sufficient condition to have a feasible solution to

$$\sum_{i=0}^{n} p_i = \sum_{i=0}^{m} q_j$$

if the transportation problem is unbalanced then dummy row or column will be created accordingly.

	1	2	…j…	n	supply
1	z_{11} x_{11}	z_{12} x_{12}	z_{1j} x_{1j}	z_{1n} x_{1n}	p_1
2	z_{21} x_{21}	z_{23} x_{22}	z_{2j} x_{2j}	z_{2n} x_{2n}	p_2
.					.

. i	z_{i1} x_{i1}	z_{i2} x_{i2}	z_{ij} x_{ij}	z_{in} x_{in}	. p_i .
m	z_{m1} x_{m1}	z_{m2} x_{m2}	z_{mj} x_{mj}	z_{mn} x_{mj}	p_m
Demand	q_1	q_2	… q_j …	q_n	

1.3 Basic definition:

- Feasible solution:

  In a transportation problem, a feasible solution refers to a set of non-negative allocations that satisfies the raw and column restrictions.

- Basic feasible solution:

  It is a feasible solution with no more that m+n-1 allocations, where m refers to number of rows and n refers to number of columns.

- Optimal solution:

  A feasible solution is called optimal if it minimizes the transportation cost and maximize the profit.

- Non-degenerate basic feasible solution:

  A basic feasible solution to a (m × n) transportation problem is called degenerate if the total number of non-negative allocations is exactly m+n-1 (i.e., number of independent constraint equations) and these allocations are in independent positions.

- Degenerate basic feasible solution:

  A basic feasible solution in which number of allocations is less than m+n-1 is called

  a degenerate basic feasible solution.

- Matix terminology:

  In transportation problem, matrix that contains squares called 'cells', when stacked form 'column' vertically and 'rows' horizontally. Unit costs are placed in each cell.

Plants	Destination1	Destination 2	Destination 3	Destination 4	Supply
A	4	3	2	11	15
B	7	6	5	8	20
demand	8	5	10	12	

2. Review of Literature:

To discuss the application of transportation problem in various practical field a linear program has been done as follows:

- Kavita Gupta (2013) in her thesis ‘On Some Aspects of Transportation Problem’ she focusses on Capacitate Transportation problem with different types of constraints and various algorithms are developed to solve them [1].
- Gaurav Gupta (2016) in his thesis ‘Transportation Problems in Intuitionistic Fuzzy Environment’ he focusses on the limitations and flaws of these existing methods are pointed out. Also, to resolve the flaws as well as to overcome the limitations of the existing method, new methods are proposed [2].
- B. D. Kavitha (2017) in her thesis ‘An economic analysis of urban public transportation in Karnataka’ she focusses on the development of roads and growth in major motor vehicles in India and Karnataka. And she analyze the growth of public and private vehicles in India and the efficiency of urban public transportation in Mysuru [3].
- Maqsood Ahmed Khan (1999) in his thesis ‘Study on multiple bottleneck transportation problem’ he focusses on the solution procedures for the Bottleneck Transportation Problem might well do extended and modified to solve the Multiple bottleneck Transportation Problem. These is a fair amount of literature on BTP is available. In this section the basic techniques, which are considered most efficient, are reviewed [4].

- Kuldeep (2019) in his thesis 'Bulk Transportation Problem and its Variants with Fuzzy Parameter' a comprehensive literature survey of the concerned work is discussed. A new algorithm is presented to obtain the minimum cost of bulk transportation which provides an easy and alternative method to the existing work in literature [5].
- In the case study (2022), "A case study of application of transportation problem in some select companies", Ahmed R and Kalita B solved the problems with the help of simplex method [6].

3. Objectives of Study:

- To collect the transportation cost from "A Case Study of Application of Transportation Problem in some select Companies".
- To minimize the transportation cost.
- To determine the required unit to be supplied to every destination

4. Methods and Materials

4.1 Materials:

In this project, the transportation cost along with demand and supply has been collected from "A Case Study of Application of Transportation Problem in Some Select Companies" done by Ahmed and Kalita in the year 2021-2022. The details of the company are:

Name of the company: Rhino research product

Year of establishment: 2005

Location: Dhubri, Assam.

It is currently a leading Manufacturer in this area. This company is a supplier of fifteen products, viz., Adapto, Pentagesic Cream, Ayush Kwath, Rhinotone Junior, Dexoliv, Triozyme, Dexoliv Forte, Tulsi Expectorant, Dexoliv Plus, Tulsi SF, Irotone, Urjashil, Rhino Oil, Vit Alka and Rhinotone.

Materials of the company 'Rhino Research Product" as stated above are recorded in the following tables:

Table 1: Demand and Supply matrix

	Chhaygaon	Dudhnoi	Boko	Nogaon	Hojai	Kharupetiya	Supply
Barpeta	12	10	17	12	10	40	101
Ghy	25	15	40	40	45	12	177
Goalpara	10	55	25	7	5	7	109
Dhubri	0	5	10	0	7	10	32
Demand	47	80	66	59	67	69	

Table 2: Cost Metrics

	Chhaygaon	Dudhnoi	Boko	Nogaon	Hojai	Kharupetiya
Barpeta	600	500	850	700	400	2300
Ghy	1250	700	2000	2100	2350	750
Goalpara	500	2700	1300	400	200	325
Dhubri	0	250	400	0	350	500

4.2 Methods:

To meet the objectives the following methods are incorporated--

- North-west corner methods of solving a transportation problem
- Least cost entry method
- Vogel's Approximation method
- Using MODI /U-V method
- Use of simplex method in Matlab2016a

4.2.1 **Mathematical formulation**:

Table 3: Allocation metrices

	Chhaygao	Dudhn	Boko	Nogao	Hojai	Kharupeti	Dummy	Supply

	n	oi		n		ya		
Barpeta	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	101
Ghy	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	177
Goalpara	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	109
Dhubri	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	32
Demand	47	80	66	59	67	69	31	

Table 4: Cost per unit:

	Chhaygaon	Dudhnoi	Boko	Nogaon	Hojai	Kharupetiya
Barpeta	600	500	850	700	400	2300
Ghy	1250	700	2000	2100	2350	750
Goalpara	500	2700	1300	400	200	325
Dhubri	0	250	400	0	350	500

5 Results and Discussion

- **Results:**

Applying the three methods of transportation method:

Since the given problem is an unbalanced, therefore a dummy column has been made with required demand.

Source	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply
O_1	600	500	850	700	400	2300	0	101
O_2	1250	700	2000	2100	2350	750	0	177
O_3	500	2700	1300	400	200	325	0	109
O_4	0	250	400	0	350	500	0	32
Demand	47	80	66	59	67	69	31	

- North-west corner method:

Source	D_1	D_2	D_3	D_4	D_5	D_6	D_7	Supply
O_1	600 (47)	500 (54)	850	700	400	2300	0	101/54/0
O_2	1250	700 (26)	2000 (66)	2100 (59)	2350 (26)	750	0	177/151/85/26/0
O_3	500	2700	1300	400	200 (41)	325 (68)	0	109/68/0
O_4	0	250	400	0	350	500 (1)	0 (31)	32
Demand	47/0	80/26/0	66/0	59/0	67/41/0	69/1	31	

$x_{11} = 47$, $x_{12} = 54$, $x_{13} = 0$, $x_{14} = 0$, $x_{15} = 0$, $x_{16} = 0$, $x_{17} = 0$, $x_{21} 0$, $x_{22} = 26$, $x_{23} = 66$, $x_{24} = 59$, $x_{25} = 26$, $x_{26} = 0$, $x_{27} = 0$, $x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 0$, $x_{35} = 41$, $x_{36} = 68$, $x_{37} = 0$, $x_{41} = 0$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 1$, $x_{47} = 31$

Total transportation cost = (600×47) + (500×54) × (700×26) + (2000×66) + (2100×59) + (2350×26) + (200×41) + (325×68) + 500 + 0 = Rs.4,21,200

- Least cost method:

Source	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply
O_1	600	500 (70)	850	700	400	2300	0 (31)	101/70
O_2	1250 (15)	700 (10)	2000 (66)	2100 (66)	2350	750 (27)	0	177
O_3	500	2700	1300	400	200 (67)	325 (42)	0	109/42/0
O_4	0 (32)	250	400	0	350	500	0	32/0
Demand	47/15	80/10	66/0	59/0	67/0	69/27/0	31/0	

$x_{11} = 0$, $x_{12} = 70$, $x_{13} = 0$, $x_{14} = 0$, $x_{15} = 0$, $x_{16} = 0$,$x_{17} = 31$, $x_{21} = 15$, $x_{22} = 10$, $x_{23} = 66$, $x_{24} = 66$,$x_{25} = 0$, $x_{26} = 27$,$x_{27} = 0$, $x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 0$, $x_{35} = 67$, $x_{36} = 42$, $x_{37} = 0$,$x_{41} = 32$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 0$, $x_{47} = 0$

Total transportation cost = (500×70) + 0 + (1250×15) + (700×10) + (2000×66) + (2100×59) + (750×27) + (200×67) + (325×42) + 0
=Rs.3,53,950

- Vogel's approximation

Source	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply	Penalty
O_1	600	500	850 (66)	700	400 (35)	2300	0	101/35/0	400/100/100/100/100/300/300/300
O_2	1250 (15)	700 (80)	2000	2100 (19)	2350 (32)	750	0 (31)	177/146/ 66/51/0	700/50/50/50/550/850/200/200
O_3	500	2700	1300	400 (40)	200	325 (69)	0	109/40/0	200/125/200/200/200
O_4	0 (32)	250	400	0	350	500	0	32/0	0/0
Demand	47/15	80/0	66/0	59/19/0	6732/0	69/0	31/0		
penalty	500/500/100/100/100	250/250/200/200/200	450/450/450	400/ 400/300/300/300/300/300.1400	150/150/200/200/200/200/200/200/2000/1900	175			

$x_{11} = 0$, $x_{12} = 0$, $x_{13} = 66$, $x_{14} = 0$, $x_{15} = 35$, $x_{16} = 0$,$x_{17} = 0$, $x_{21} = 15$, $x_{22} = 80$, $x_{23} = 0$, $x_{24} = 19$,$x_{25} = 32$, $x_{26} = 0$,$x_{27} = 31$, $x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 40$, $x_{35} = 0$, $x_{36} = 69$, $x_{37} = 0$,$x_{41} = 32$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 0$, $x_{47} = 0$

Total transportation cost = (850×66) + (400×35) + (1250×15) + (700×80) + (2100×19) + (2350×32) + 0 + (400×40) + (325×69) + 0

= Rs.2,98,375

- In order to determine the optimality of the methods, we use U-V method/MODI method

Taking the result of vogel's approximation method:

Source	D_1	D_2	D_3	D_4	D_5	D_6	D_7	supply
O_1	600	500	850 (66)	700	400 (35)	2300	0	101/35/0
O_2	1250 (15)	700 (80)	2000	2100 (19)	2350 (32)	750	0 (31)	177/146/ 66/51/0
O_3	500	2700	1300	400 (40)	200	325 (69)	0	109/40/0
O_4	0 (32)	250	400	0	350	500	0	32/0
Demand	47/15	80/0	66/0	59/19/0	6732/0	69/0	31/0	

Total transportation cost=Rs.2, 98, 375

The U-V matrix is as under:

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	**1250 (15)**	**700 (80)**	2000	**2100 (19)**	**2350 (32)**	750	**0 (31)**
U_3	500	2700	1300	**400 (40)**	200	**325 (69)**	0
U_4	**0 (32)**	250	400	0	350	500	0

To apply MODI method, it should satisfy the condition:

m+n-1= number of allocations made, where 'm' refers to number of rows and 'n' refers to number of columns of a transportation problem.

Here, m+n-1=10, number of allocations=10, therefore, m+n-1= number of allocations made now, finding u and v from the allocated cells

assuming, $u_{1=}0$

$u_1+v_3 = 850 \Rightarrow v_3=850$, $u_1+v_5=400 \Rightarrow v_5=400$, $u_2+v_5=2350 \Rightarrow u_2=1250$

Similarly proceeding, we get

$v_1= -700$, $v_2= - 1250$, $v_4= 150$, $v_7= -1950$, $u_3=250$, $v_6=75$, $u_4=700$

to determine the penalties for the unallocated cells,

$p_{ij} = u_i+v_j - c_{ij}$

$p_{11}= -1300$

$p_{12}= -1750$

$p_{14}= -550$

$p_{16}= -2225$

$p_{17}=-1950$

$p_{23}= 800$

$p_{26}= 1275$

$p_{31}= -950$

$p_{32}= -3700$

$p_{33}=-200$

$p_{35}= 450$

$p_{37} =325$

$p_{43}= 1150$

$p_{44}= 850$

$p_{45}=750$

$p_{46=}275$

$p_{47}= -1250$

$p_{42}=-800$

$p_{37} =-1700$

If our penalty is zero or negative, then the optimality is reached and this is the final answer. But here we are getting some positive penalties. Therefore, we construct a loop starting from the highest cell containing positive penalty in such a way that the right- angle turn will be taken at the allocated cell or new basic cell.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	**1250 (15)**	**700 (80)**	2000	**2100 (19) -**	**2350 (32)**	750 +	**0 (31)**
U_3	500	2700	1300	**400 (40)** +	200	**325 (69)**	0
U_4	**0 (32)**	250	400	0	350	500	0

Taking the minimum from the negative cell of the loop and subtracting it from the negative cells and adding it to the positive cells, we get

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	**1250 (15)**	**700 (80)**	2000	**2100**	**2350 (32)**	**750 (19)**	**0 (31)**
U_3	500	2700	1300	**400 (59)**	200	**325 (50)**	0
U_4	**0 (32)**	250	400	0	350	500	0

Again, applying the MODI method while checking the condition and after finding the u and v for the allocated cells, we get some positive penalties. those are:

$P_{23}=800$, $p_{31}=150$, $p_{33}=500$, $p_{35}=1550$, $p_{36}=150$, $p_{43}=1150$, $p_{45}=850$

taking the highest positive cell and creating a close loop, we get

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	**1250 (15)**	**700 (80)**	2000	**2100**	**2350 (32)** -	**750 (19)** +	**0 (31)**
U_3	500	2700	1300	**400 (59)**	200 +	**325 (50)** -	0
U_4	**0 (32)**	250	400	0	350	500	0

Following the same procedure, we get

	V_1	V_2	V_3	V_4	V_5	V_6	V_7

U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	**1250 (15)**	**700 (80)**	2000	2100	2350	**750 (51)**	**0 (31)**
U_3	500	2700	1300	**400 (59)**	**200 (32)**	**325 (18)**	0
U_4	**0 (32)**	250	400	0	350	500	0

Again, applying MODI method, while checking the condition, we get some positive penalties-

P_{11}=525, P_{31}=1050, P_{36}=325

Taking the cell with most positive penalty

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	**1250 (15)** -	**700 (80)**	2000	**2100**	**2350**	**750 (51)** +	**0 (31)**
U_3	500 +	2700	1300	**400 (59)**	200 **(32)**	**325 (18)** -	0
U_4	**0 (32)**	250	400	0	350	500	0

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600	500	**850 (66)**	700	**400 (35)**	2300	0
U_2	1250	**700 (80)**	2000	2100	2350	**750 (66)**	**0 (31)**
U_3	**500 (15)**	2700	1300	**400 (59)**	**200 (32)**	**325 (3)**	0
U_4	**0 (32)**	250	400	0	350	500	0

Again , applying MODI method, we got some positive penalties-

P_{11}=425, P_{12}=75, P_{14}=225

Now, making the loop

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
U_1	600 +	500	**850 (66)**	700	**400 (35)**	2300	0
	V_1	V_2	V_3	V_4	V_5 -	V_6	V_7
U_2	1250	**700 (80)**	2000	2100	2350	**750 (66)**	**0 (31)**
U_1	**600 (15)**	500	**850 (66)**	700	**400 (20)**	2300	0
U_3	**500 (15)**	2700	1300	**400 (59)**	**200 (32)**	**325 (3)**	0
U_2	1250 -	**700 (80)**	2000	2100	2350 +	**750 (66)**	**0 (31)**
U_4	**0 (32)**	250	400	0	350	500	0
U_3	500	2700	1300	**400 (59)**	**200 (47)**	**325 (3)**	0
U_4	**0 (32)**	250	400	0	350	500	0

Applying MODI method, we don't get any more positive penalty, therefore it produces the optimal solution.

$x_{11} = 15$, $x_{12} = 0$, $x_{13} = 66$, $x_{14} = 0$, $x_{15} = 20$, $x_{16} = 0$, $x_{17} = 0$, $x_{21} = 0$, $x_{22} = 80$, $x_{23} = 0$, $x_{24} = 0$, $x_{25} = 0$, $x_{26} = 66$, $x_{27} = 31$, $x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 59$, $x_{35} = 47$, $x_{36} = 3$, $x_{37} = 0$, $x_{41} = 32$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 0$, $x_{47} = 0$

Total transport cost =Rs. 2,12,575

- **Using simplex method**:

Minimum Cost = $600x_{11} + 500x_{12} + 850x_{13} + 700x_{14} + 400x_{15} + 2300x_{16} + 0.x_{17} + 1250x_{21} + 700x_{22} + 2000x_{23} + 2100x_{24} + 2350x_{25} + 750x_{26} + 0.x_{27} + 500x_{31} + 2700x_{32} + 1300x_{33} + 400x_{34} + 200x_{35} + 325x_{36} + 0.x_{37} + 0.x_{41} + 250x_{42} + 400x_{43} + 0.x_{44} + 350x_{45} + 500x_{46} + 0.x_{47}$

- Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \le 101$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \le 177$$

$$x_{31} + x_{32} + x_{33} + x_{44} + x_{35} + x_{36} + x_{17} \leq 109$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} \leq 32$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 47$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 80$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 66$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 59$$

$$x_{15} + x_{25} + x_{35} + x_{45} = 67$$

$$x_{16} + x_{26} + x_{36} + x_{46} = 69$$

$$x_{17} + x_{27} + x_{37} + x_{47} = 31$$

x_{11} , x_{12} , x_{13} , x_{14} , x_{15} , x_{16} , x_{17} , x_{21} , x_{22} , x_{23} , x_{24} , x_{25} , x_{26} , x_{27} , x_{31} , x_{32} , x_{33} , x_{34} , x_{35} , x_{36}, x_{37} , x_{41} , x_{42} , x_{43} , x_{44} , x_{45} , x_{46}, x_{47} , ≥ 0

Optimization terminated at

x = 21.8496, 0.0000, 66.0000, 0.0000, 13.1504, 0.0000, 0.0000, 0.0000, 80.0000, 0.0000, 0.0000, 0.0000, 66.0000, 31.0000, 0.0000, 0.0000, 0.0000, 52.1504, 53.8496, 3.0000, 0.0000, 25.1504, 0.0000, 0.0000, 6.8496, 0.0000, 0.0000, 0.0000.

Z = Rs.2,12,580.

3.1 **Discussion**:

Using North-West corner method, the following results are obtained:

$x_{11} = 47$, $x_{12} = 54$, $x_{13} = 0$, $x_{14} = 0$, $x_{15} = 0$, $x_{16} = 0$, $x_{17} = 0$, $x_{21}0$, $x_{22} = 26$, $x_{23} = 66$, $x_{24} = 59$, $x_{25} = 26$, $x_{26} = 0$, $x_{27} = 0$, $x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 0$, $x_{35} = 41$, $x_{36} = 68$, $x_{37} = 0$, $x_{41} = 0$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 1$, $x_{47} = 31$

Total transportation cost = Rs.4,21,200

In this method, the allocated cells are: $C_{11} = 47$, $C_{12} = 54$, $C_{22} = 26$, $C_{23} = 66$, $C_{24} = 59$,$C_{25} = 26$, $C_{35} = 41$, $C_{36} = 68$, $C_{46} = 1, C_{47} = 31$ where C_{ij} =Cell containing ith row and jth column

Using least cost entry method, the results obtained are:

$x_{11} = 0$, $x_{12} = 70$, $x_{13} = 0$, $x_{14} = 0$, $x_{15} = 0$, $x_{16} = 0$,$x_{17} = 31$, $x_{21} = 15$, $x_{22} = 10$, $x_{23} = 66$, $x_{24} = 66$,$x_{25} = 0, x_{26} = 27$,$x_{27} = 0, x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 0$, $x_{35} = 67$, $x_{36} = 42, x_{37} = 0$,$x_{41} = 32$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 0, x_{47} = 0$

Total transportation cost = (500×70) + 0 + (1250×15) + (700×10) + (2000×66) + (2100×59) + (750×27) + (200×67) + (325×42) + 0
=Rs.3,53,950

In this method, the allocated cells are: $C_{12} = 70$, $C_{17} = 31$, $C_{21} = 15$, $C_{22} = 10$, $C_{23} = 66$, $C_{24} = 66$,$C_{26} = 27$, $C_{35} = 67$, $C_{36} = 42$, $C_{41} = 32$ where C_{ij} =Cell containing ith row and jth column

Using Vogel's approximation method, the results obtained are:

$x_{11} = 0$, $x_{12} = 0$, $x_{13} = 66$, $x_{14} = 0$, $x_{15} = 35$, $x_{16} = 0$,$x_{17} = 0$, $x_{21} = 15$, $x_{22} = 80$, $x_{23} = 0$, $x_{24} = 19$,$x_{25} = 32, x_{26} = 0$,$x_{27} = 31, x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 40$, $x_{35} = 0$, $x_{36} = 69, x_{37} = 0$,$x_{41} = 32$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 0, x_{47} = 0$

Total transportation cost = (850×66) + (400×35) + (1250×15) + (700×80) + (2100×19) + (2350×32) + 0 + (400×40) + (325×69) + 0 = Rs.2,98,375

In this method, the allocated cells are: $C_{13} = 66$, $C_{15} = 35$, $C_{21} = 15$, $C_{22} = 80$, $C_{24} = 19$,$C_{25} = 32$, $C_{27} = 31, C_{34} = 40$, $C_{36} = 69$, $C_{41} = 32$ where C_{ij} =Cell containing ith row and jth column

Using U-V method, results obtained are:

$x_{11} = 15$, $x_{12} = 0$, $x_{13} = 66$, $x_{14} = 0$, $x_{15} = 20$, $x_{16} = 0$,$x_{17} = 0$, $x_{21} = 0$, $x_{22} = 80$, $x_{23} = 0$, $x_{24} = 0$,$x_{25} = 0, x_{26} = 66$,$x_{27} = 31, x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} =$

59 , $x_{35} = 47$, $x_{36} = 3, x_{37} = 0$, $x_{41} = 32$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 0$, $x_{45} = 0$, $x_{46} = 0, x_{47} = 0$

Total transport cost =Rs. 2,12,575

In this method, the allocated cells are: $C_{11} = 15$, $C_{13} = 66$, $C_{22} = 80$, $C_{26} = 66$, $C_{27} = 31$, $C_{34} = 59$, $C_{36} = 3$, $C_{41} = 32$ where C_{ij} =Cell containing ith row and jth column

Using simplex method with 'linprog' solver of Matlab2016a, the results obtained are:

$x_{11} = 22$, $x_{12} = 0$, $x_{13} = 66$, $x_{14} = 0$, $x_{15} = 13$, $x_{16} = 0$, $x_{17} = 0$, $x_{21} = 0$, $x_{22} = 80$, $x_{23} = 0$, $x_{24} = 0$, $x_{25} = 0$, $x_{26} = 66$, $x_{27} = 31$, $x_{31} = 0$, $x_{32} = 0$, $x_{33} = 0$, $x_{34} = 52$, $x_{35} = 54$, $x_{36} = 3, x_{37} = 0$, $x_{41} = 25$, $x_{42} = 0$, $x_{43} = 0$, $x_{44} = 7$, $x_{45} = 0$, $x_{46} = 0, x_{47} = 0$

The transportation cost = Rs.2,12,580.

In this method, the allocated cells are: $C_{11} = 22$, $C_{13} = 66$, $C_{15} = 13$, $C_{22} = 80$, $C_{26} = 66$, $C_{27} = 31$, $C_{34} = 52$, $C_{35} = 54$, $C_{36} = 3$, $C_{41} = 25$, $C_{44} = 7$ where C_{ij} =Cell containing ith row and jth column

6 Conclusion:

In this project, one company is considered for studying application of transportation problem. The following conclusion is given-

- To solve the problem simplex method is used
- Also, three methods of transportation problem are used
- To check the optimality of Vogel's approximation method, MODI method is used.
- By simplex method, we get total transportation cost = Rs.2,12,580.
- By methods of transportation, total transportation cost = Rs.2,12,575
- In North-west corner method, the allocated cells are: (1,1), (1,2), (2,2), (2,3), (2,4), (2,5), (3,5), (3,6), (4,6) and (4,7).
- In least cost entry method, the allocated cells are: (1,2), (1,7), (2,1), (2,2), (2,3), (2,4), (2,6), (3,5), (3,6) and (4,1).

- In Vogel's approximation method, the allocated cells are: (1,3), (1,5), (2,1), (2,2), (2,4), (2,5), (2,7), (3,4), (3,6) and (4,1).
- In MODI method, the allocated cells are: (1,1), (1,3), (2,2), (2,6), (2,7), (3,4), (3,6) and (4,1).
- In Simplex method, the allocated cells are: (1,1), (1,3), (1,5), (2,2), (2,6), (2,7), (3,4), (3,5), (3,6), (4,1) and (4,4).

Future Scope of Study:

- Extend the region
- Extend the work to some other company

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